Using Mathematical and Scientific Markup as an Approach to Model Specification

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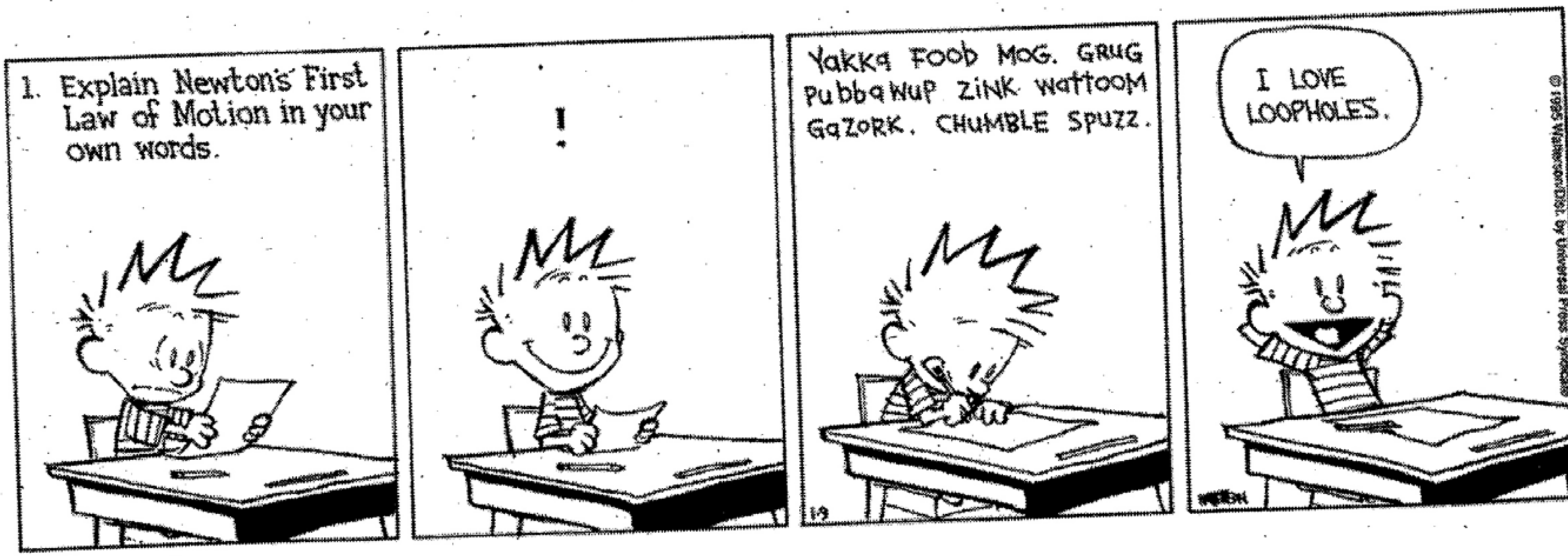
1. Explain Newton's First Law of Motion in your own words.
!
Yakka Foob Mog. Grug Pubbawup Zink wattoom Gazork. Chumble Spuzz.
I LOVE LOOPHOLES.
© 1995 Watterson/Dist. by Universal Press Syndicate

What are we talking about?

- *“The Web would make a dandy blackboard if only we could scribble an equation*”
  - “Writing Math on the Web”, Brian Hayes, American Scientist, March-April 2009 Vol 97, No. 2
- *Mars Climate Orbiter*
  - Lost, 23 Sept 1999. Confusing pounds force (ground control software) with newtons (spacecraft software).
- *Korean Air, cargo flight 6316*
  - Fatal crash, 15 April 1999 from Shanghai to Seoul, confusing metres (tower) and feet (altimeter)
- *The Gimli Glider*
  - 23 July 1983, out of fuel at 41,000 ft in an Air Canada Boeing 767-200 jet, confusing litres for gallons

Metadata and Ontologies

- Provide “Content Descriptors”
- Refers to agreed-upon semantics
- An ontology provides a framework for metadata
- Supports:
  - Documentation
  - Registries and Repositories for re-use
  - Search and retrieval by semantic content
  - Use with Computer Algebra Solvers
- Ontology -> Metadata -> Markup language

MathML: Differential Equations

$$\nabla^2 G - \frac{\nabla\rho}{\rho}\bullet\nabla G + \frac{1}{c^2}\frac{\partial^2 G}{\partial t^2} = -\delta(r - r')\delta(t - t')$$

```
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   <divergence/>
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   </apply>
</apply>
```
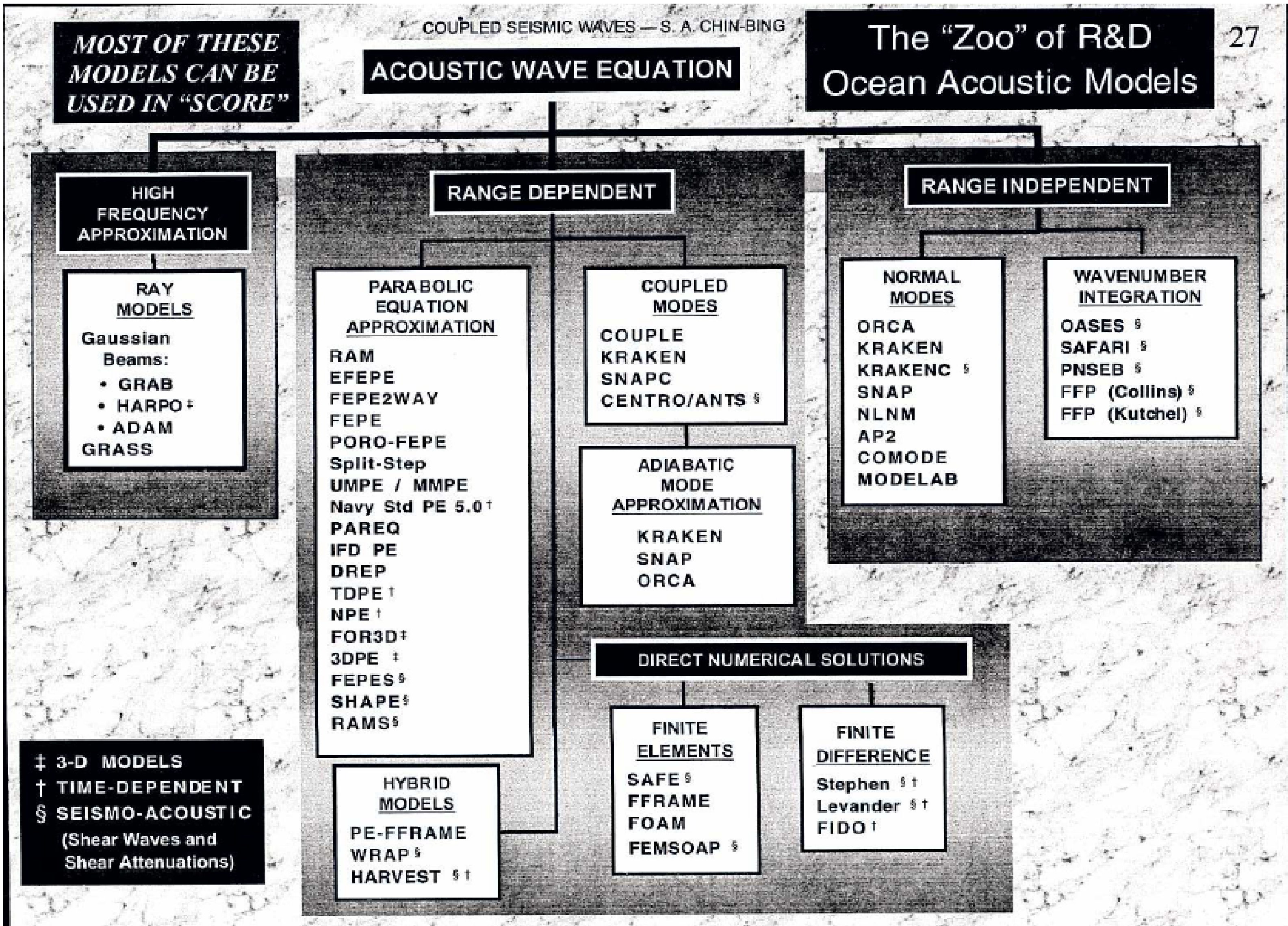

The Acoustic Model Zoo



A Physics-Based Model Ontology Layercake

- The thing modeled
- The Physics
- Language of physics
- The easier solution
- Let’s get an answer!
- The next guy’s grid

Physical object

Physical concepts

Mathematical Expr.

Mathematical approx

Discretized approx

Interpolation

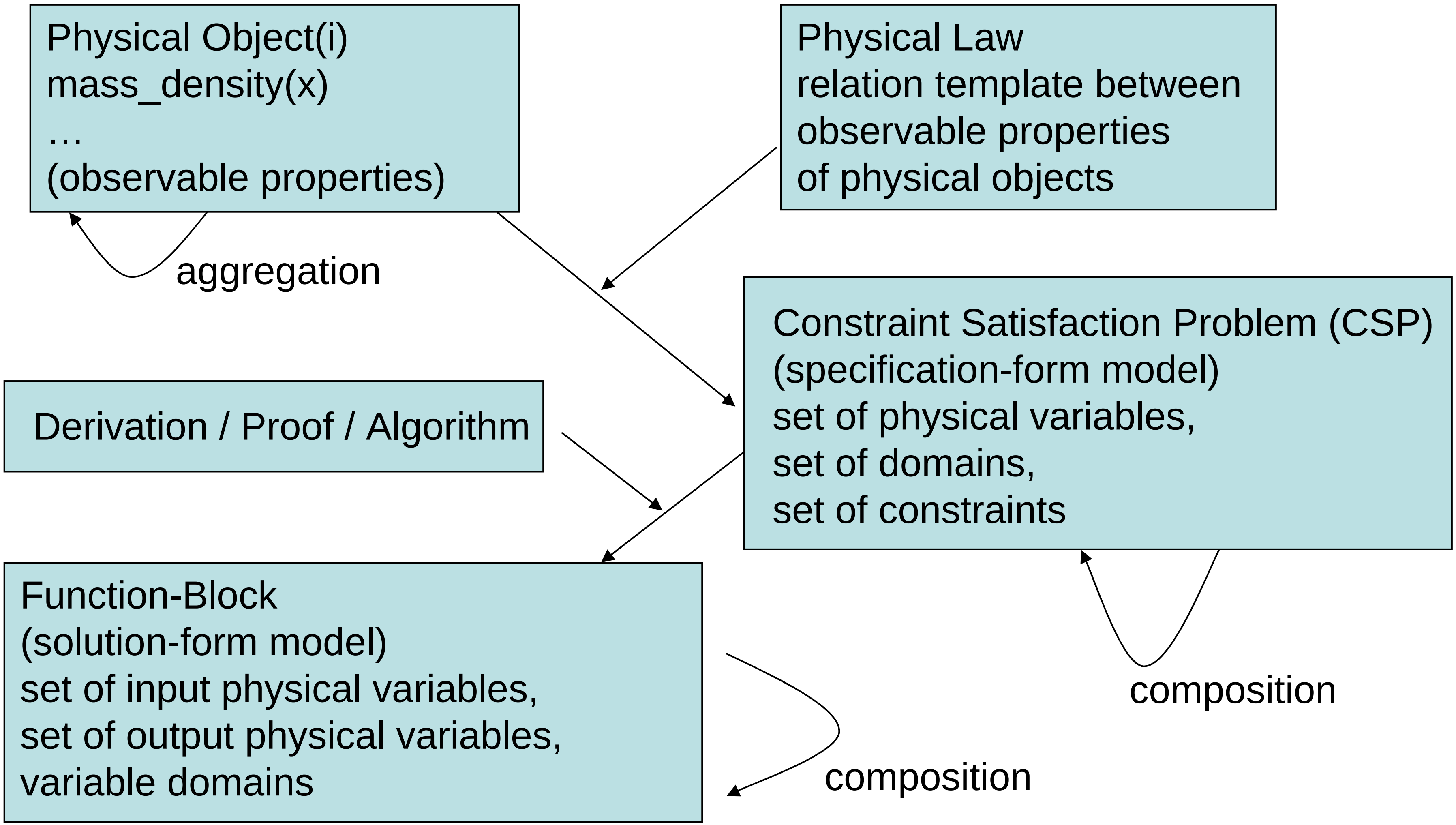
Each layer to layer, downward transition is **informal**, **one-to-many**

Can we infer the Physical Concept from the last layer? **No!**

Economic Analysis

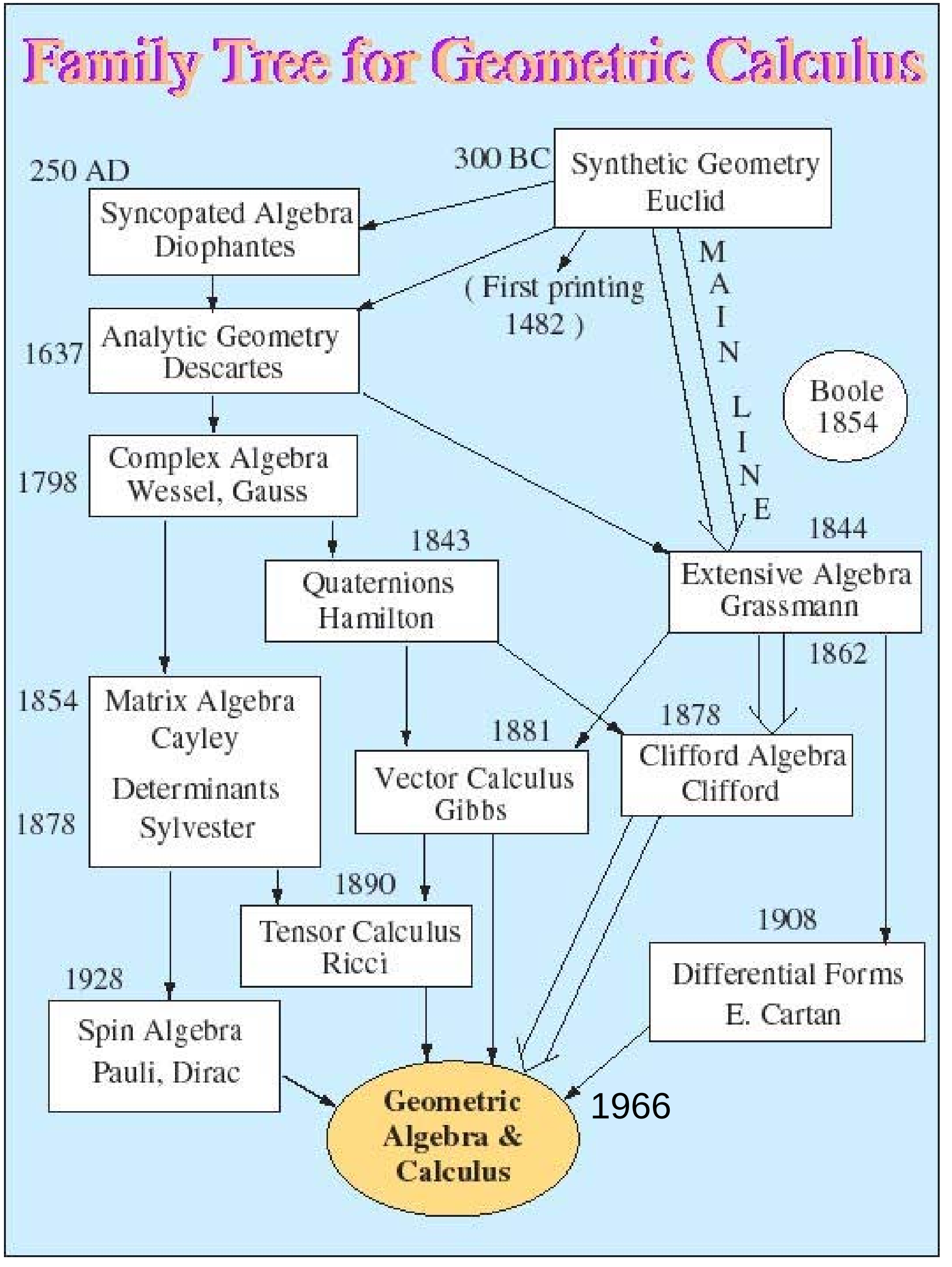
- Adam Smith’s Division of Labor (1776)
  - Dynamic engine of economic progress
  - 1 master “pin-maker” vs. many sub-specialists
    - “making a pin is, in this manner, divided into about eighteen distinct operations”
  - 100-1000 fold improvement in productivity
- We rely too heavily on master modelers
- Standardization: Applications or Interchange?
- Unambiguous interchange required
  - For effective division of labor in modeling
  - For effective communication between models

Some Semantic Concept Classes for Physics Markup Elements



Hestenes' Pedigree* of Geometric Algebra

- GA unifies several disparate notations used in physics
- GA undergoing continuing development for application to physics



*http://modelingnts.la.asu.edu/html/evolution.html

How to Reduce Ambiguity

- Be Formal (Use Math)
- Use Standards
  - Community accepted concepts, definitions
  - Open is better
- Document Assumptions
  - For easier understanding
  - For non-experts also
- Compatible with Computer Applications
  - Practical criterion: Usable in CA Systems

Web-Based = XML Applications

- XML is the emerging baseline for knowledge representation on the Web
- Content MathML and OpenMath are XML applications for specification of mathematical content
- OMDoc and DocBook-MathML are XML applications capable of representing mathematical documents

MathML

- Two Flavors specified
  - Presentation MathML and Content MathML
- Provides concept names for basic math
- Provides a construct for extension
- Many current web-browsers display it
- Reasonably mature W3C Recommendation
  - (v.2 going on v.3)
  - OpenMath being merged with MathML

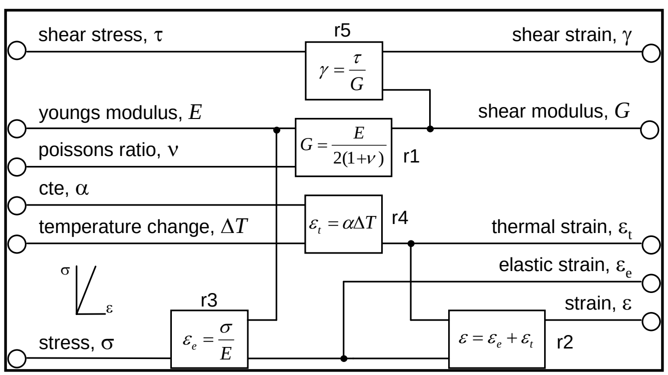
COB-based Libraries of Analysis Building Blocks (ABBs)

Material Model and Continuum ABBs - Constraint Schematic-S

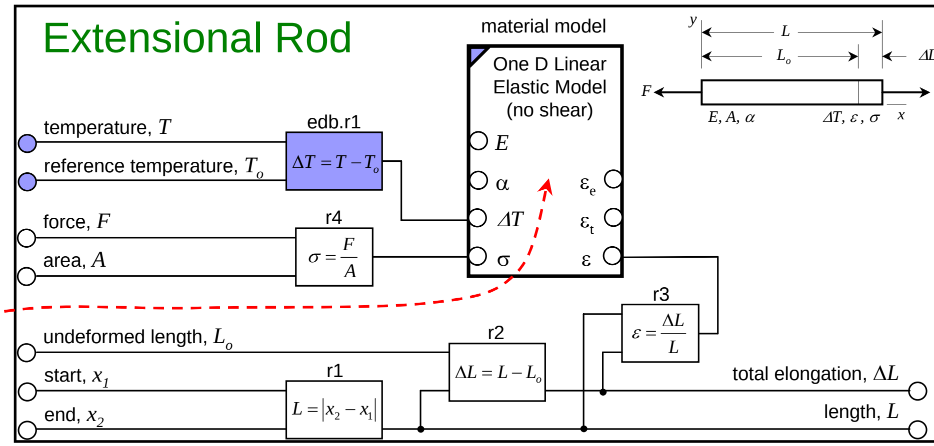
Continuum ABBs

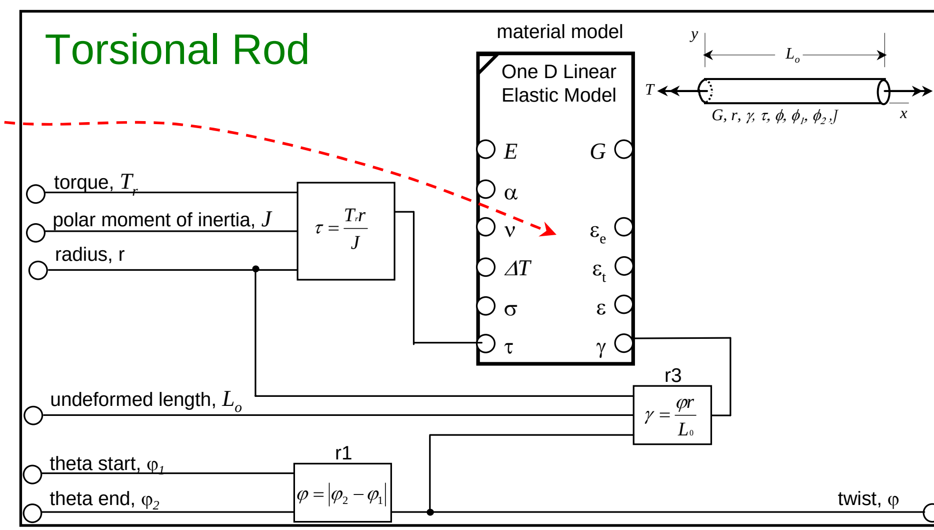
Material Model ABB

1D Linear Elastic Model



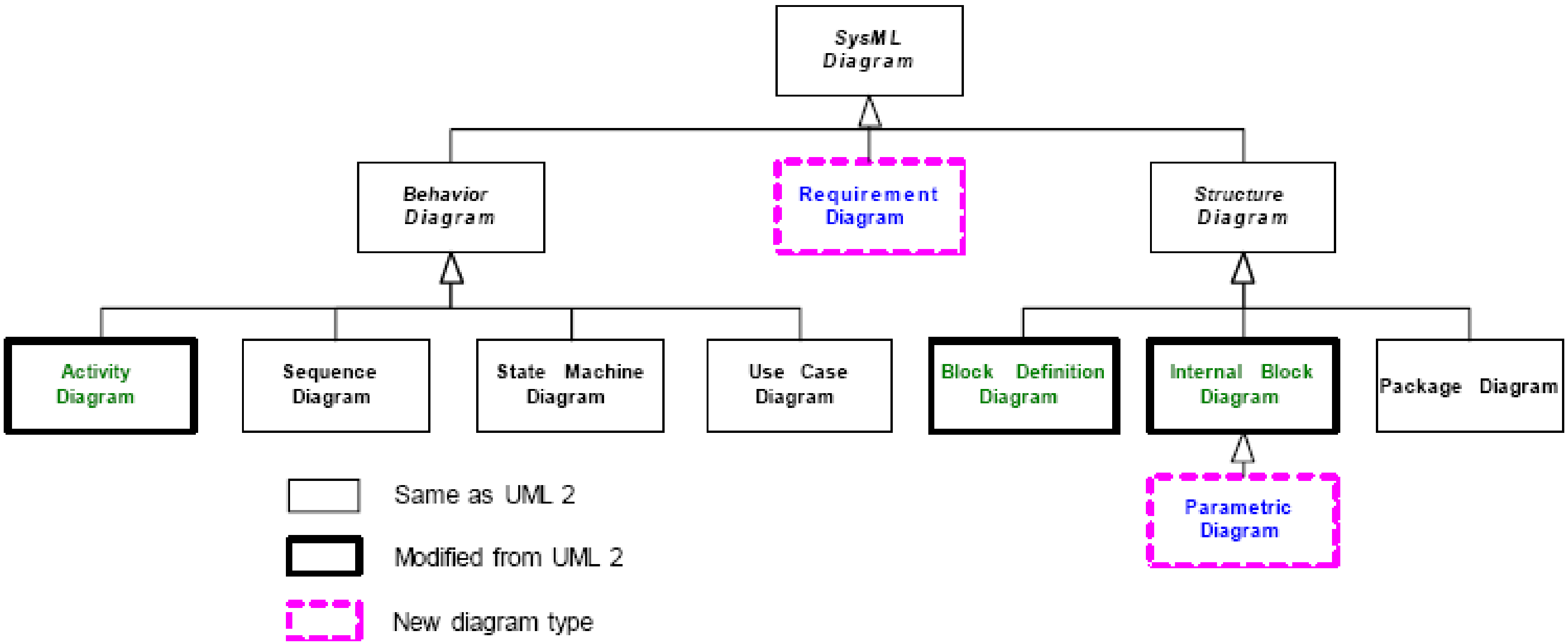
modular
re-usage





Prof. Russell Peak, GA Tech
http://eislab.gatech.edu/projects/nasa-ngcobs/ -2005-06-01

SysML Diagram Taxonomy



SysML Parametric Diagram (New)

- Used to express constraints (equations) between value properties
  - Provides support for engineering analysis (e.g., performance, reliability)
- Constraint block captures equations
  - Expression language can be formal (e.g., MathML, Object Constraint Language) or informal
  - Computational engine is defined by applicable analysis tool and not by SysML
- Parametric diagram represents the usage of the constraints in an analysis context
  - Binding of constraint usage to value properties of blocks (e.g., vehicle mass bound to F= m × a)
- Parametrics Enable Integration of Engineering Analysis with Design Models

Documentation

- OMDoc appears very promising
  - Principals receptive to collaboration
- OMDoc built on MathML and OpenMath
  - Based on “modules” of elements: DOC, Dublin Core, Creative Commons, Content MathML, OpenMath, Math Text, Mathematical Statements, Semantic Reference, Abstract Data Types, Proofs, Complex Theories, …
- “Content Dictionary” based modules
  - Physical observables, physical constants, physical objects, physical laws

Benefits

- Support for Warfighter in building systems, simulations, wargames, analyses
- Open access to models by non-experts
  - Search, retrieval, and understanding
- Automation in model documentation
- Decision aids for simulation builders
- Multi-Physics Test and Evaluation
- Approach to true composability

Backup Slides

Physical Objects

- A tuple of observable attributes
- A Physical Object may be an aggregate
  - Primitive Physical Objects
  - Physical Objects having Physical Objects as parts
- No “Is-A” taxonomies
  - No physical semantics of consequence apparent
  - Taxonomies perhaps for other considerations

Physical Observables

- A symbol representing a measurable physical quantity
  - May be a function, e.g., over space-time
  - Value is a product of two factors
- A “unit” factor
  - With vector-space properties
- A “Geometric” or “Spatial” factor
  - Scalar, Vector, Tensor, etc.
  - Element of a Geometric Algebra (Clifford Algebra)

Physical Laws

- Applied to Physical Objects
  - Individually and to interactions between
- Results in constraint relations
  - Between physical objects via their attributes
- Applied at modeler’s discretion
- Correspond to “canonical” laws, equations

Constraint Satisfaction Problem (CSP)

- A standard form for defining a mathematical problem requiring a solution
  - A modern computer science subject area
- For physics, domains generally defined over Reals
  - And complex, vectors, etc., rather than integers
- Defines a “specification level”, packaged description / definition of a model
  - Constraints are “requirements”, declaratively expressed
- Reflects practice of physics-based modeling
  - Differential equations with boundary conditions

Function Block

- Most common representation of a software module.
- Assignment
  - Combines equivalence and precedence constraints
  - Naturally maps initial conditions to future behavior
- Defines a “solution level”, packaged description / definition of a model
  - Constraints are no longer explicitly expressed
- May be composed
  - But must observe precedence constraints on variables

OMG SysML Partners

- Industry
  - American Systems, EADS Astrium, BAE SYSTEMS, Boeing, Deere & Company, Eurostep, Israel Aircraft Industries, Lockheed Martin, Motorola, Northrop Grumman, oose.de, Raytheon, THALES
- Government
  - DoD/OSD, NASA/JPL, NIST
- Vendors
  - Artisan, Ceira, Gentleware, IBM/Rational, I-Logix, PivotPoint Technology, Popkin, Project Technology, 3SL, Telelogic, Vitech
- Liaisons
  - AP-233, CCSDS, EAST, INCOSE, Rosetta

Model Characteristics Summarized